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Comments on the Monkman–Grant and the modified Monkman–Grant relationships

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The Monkman–Grant and the modified Monkman–Grant relationships are shown to be a consequence of a simple mathematical analysis of the strain rate–time curves and of a smoothing-out of the scattering in the results produced by the double-logarithmic representation. The analysis is applied to experimental data obtained in Zircaloy-4 and in stainless steel AISI 304.

1. Introduction

According to Monkman and Grant [1] a close relationship exists between minimum creep rate and time to fracture for several metals and alloys. This relationship is described by the so-called Monkman–Grant (MGR) equation

$$\log t_f + m \log \dot{\epsilon}_{min} = C \qquad (1)$$

where t_f is the time to fracture, $\dot{\epsilon}_{min}$ is the minimum creep rate and m, C are assumed to be constant.

From an analysis of the experimental data obtained on a number of metals and alloys, however, Dobeš, and Milička [2] have shown that the scatter in the experimental results can be reduced by using the relationship

$$\log (t_f/\epsilon_f) + m' \log \dot{\epsilon}_{min} = C' \qquad (2)$$

where ϵ_f is the strain to failure and m', C' are constants. Equation 2 is the so-called modified Monkman–Grant (MMGR) relationship. Toscano and Boček [3] have recently applied Equations 1 and 2 to constant load creep experiments performed on Zircaloy-2, Zircaloy-4 and stainless steel AISI 304 at high temperatures. m and C were found to be dependent on stress and temperature but m' was close to 1, as observed in [2] for different materials, and C', according to these authors, was independent of stress and temperature. Furthermore, Toscano and Boček have claimed to derive Equation 2 from an atomistic model of intergranular fracture due to Edward and Ashby [4].

It is the purpose of this paper to show that Equations 1 and 2 are a consequence of a simple mathematical analysis of the strain rate–time curves and of a smoothing-out of the scattering in the results produced by the log–log representation. Finally, the analysis will be applied to actual experimental data obtained on Zircaloy-4 and on SS AISI 304.

2. Theory

Fig. 1 shows schematically typical curves of strain rate plotted against time obtained during creep experiments at constant load and at constant temperature. The analysis that follows will be also valid if the experiments are conducted at constant stress.

The strain to fracture is given by

$$\epsilon_f(\sigma, T, S) = \int_0^{t_f(\sigma, T, S)} \dot{\epsilon}(\sigma, T, S, t)\, dt \qquad (3)$$

where T is the absolute temperature, σ is the initial applied stress and S is some parameter that characterized the initial structure of the specimen. According to the first mean-value theorem of a definite integral [5] Equation 3 can be expressed as

$$\epsilon_f(\sigma, T, S) = t_f(\sigma, T, S)\bar{\dot{\epsilon}}(\sigma, T, S) \qquad (4)$$

where $\bar{\dot{\epsilon}}(\sigma, T, S)$ is some strain rate in the interval

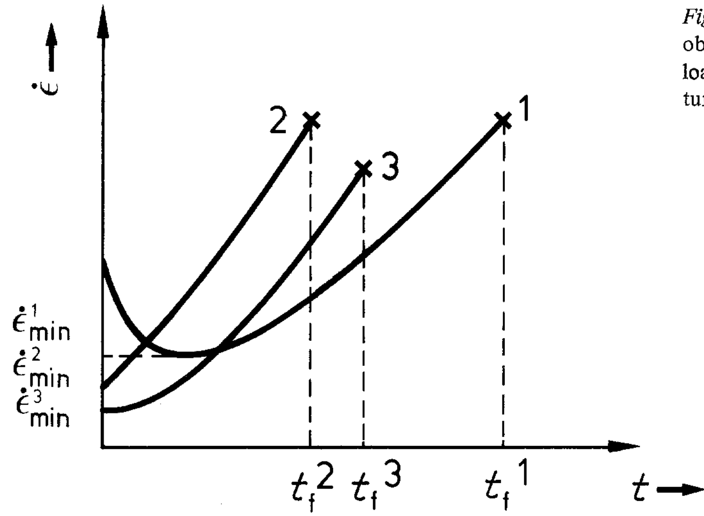

Figure 1 Typical strain rate–time curves obtained during creep experiments at constant load. σ is the initial stress and T the temperature. $\dot{\epsilon}_{\min}$ is the minimum creep rate.

0 to t_f. In what follows, the independent variables will not be specified to avoid confusion.

On assuming

$$\bar{\dot{\epsilon}} = K\dot{\epsilon}_{\min} \quad (5)$$

where K is a constant, Equation 4 is reduced to

$$\epsilon_f = K\dot{\epsilon}_{\min}t_f \quad (6)$$

If this equation is written as

$$\log t_f + \log \dot{\epsilon}_{\min} = \log (\epsilon_f/K) \quad (7)$$

the MGR relationship is obtained with $m = 1$ and $C = \log(\epsilon_f/K)$. If Equation 6 is written as

$$\log (t_f/\epsilon_f) + \log \dot{\epsilon}_{\min} = -\log K \quad (8)$$

then, the MMGR relationship is obtained with $m' = 1$ and $C' = -\log K$.

Equation 6 shows that the area under the $\dot{\epsilon}$–t curve, as expressed by Equation 3, is replaced by an area proportional to that of the rectangle of sides $\dot{\epsilon}_{\min}$ and t_f. This is illustrated schematically in Fig. 2. Furthermore, from Equation 6 it is seen that $K \geqslant 1$, so that $C' \leqslant 1$ in the MMGR relationshop when $m' = 1$.

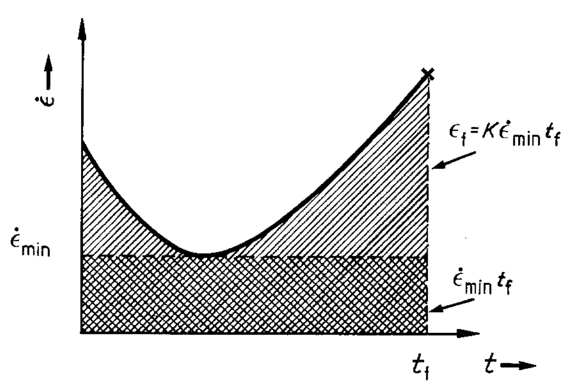

Figure 2 The integral under the $\dot{\epsilon}$–t curve, indicated by the hatched area, is expressed as a multiple of the cross-hatched area of the rectangle.

In summary, it is proposed that both the MGR and the MMGR relationships have mainly a geometrical meaning and are a consequence of Equation 6. This will be confirmed by considering several sets of experimental data.

3. Applications

Dobeš and Milička [2] have plotted several sets of data, obtained on various alloys, according to Equations 1 and 2. As pointed out by these authors, however, Equation 1 leads to a wide scatter in the data and will not be considered in detail.

If Equations 7 and 8 hold, then both the MGR and the MMGR plot should give $m = m' = 1$. This is the case for the data of Alloy A5 shown in Figs. 1 and 3 of [2], where a not too wide scatter was found for the MGR plot. In fact, the values $m = 0.95$ and $m' = 0.97$ were reported. In addition, from Table II of [2] it is seen that m' for the MMGR plot was found to be very close to 1, as suggested by Equation 8. The values for m in the MGR plot were also found to be fairly close to 1, as suggested by Equation 7, but they should be considered with caution since ϵ_f is included in the pseudo-constant of the MGR relationship. Then, a wide scatter in the MGR plot will be observed, except when ϵ_f does not change considerably with stress and temperature. Furthermore, from Equations 7 and 8 $C - C' = \log \epsilon_f$. Unfortunately, the strains to fracture were not reported in [2] but a calculation of ϵ_f using the C and C'

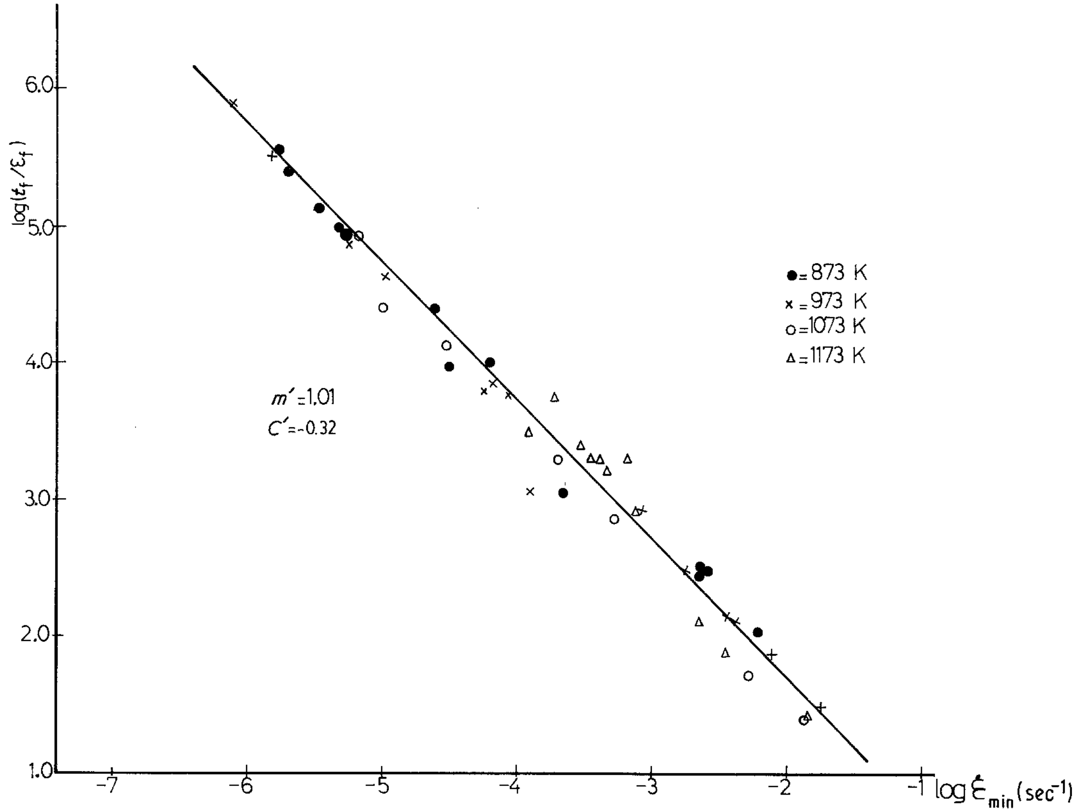

Figure 3 MMGR plot for constant load creep data obtained in Zircaloy-4.

values given in Table II of [2] give values between 0.1 and 0.3 which are reasonable numbers. Next, some creep data obtained at constant load both in Zircaloy-4 and SS AISI 304 will be considered. As shown by Toscano and Boček [3] these data give m and C values that depend on stress and temperature so that the MGR representation will not be considered. The corresponding ϵ_f, t_f and $\dot{\epsilon}_{min}$ values and the composition of the alloys are given by Petersen and Koutsandreou [6].

Fig. 3 shows a MMGR plot of the data obtained in Zircaloy-4 at temperatures between 873 and 1173 K and initial stresses between about 3 and 120 MPa. The straight line shown in the figure leads to $m' = 1.01$ and $C' = -0.32$.

Once m' is known, C' can be evaluated from Equation 2 for each individual t_f, ϵ_f and $\dot{\epsilon}_{min}$ set of data. The results are shown in Fig. 4a where C' is plotted as a function of the initial stress, for different temperatures. It is clear that C' is not constant but depends both on stress and temperature. The broken straight line shown in the figure, for example, illustrates roughly the stress dependence of C' for the data obtained at 1173 K. Furthermore, since the slope and intercept of the straight line drawn through the data points of Fig. 3 depend somewhat on the criterion used, a calculation of C' can be made for $m' = 1$, so that the result will not depend on how the straight line is drawn. C' calculated in this way is shown in Fig. 4b as a function of the initial stress and for different temperatures. Again, it is clear that, in general, C' depends both on stress and on temperature. This is illustrated by the broken straight line shown in the figure, given roughly the stress dependence of C' for the data obtained at 1173 K.

Fig. 5 shows a MMGR plot of similar data obtained in solution annealed SS AISI 304, at temperatures between 823 and 1023 K and stresses between about 70 and 370 MPa. The straight line shown in the figure gives $m' = 0.91$ and $C' = 0.17$. C' as calculated from Equation 2 for $m' = 0.91$ is shown in Fig. 6a, as a function of stress and for the different temperatures. C' calculated with $m' = 1$ is shown, as a function of the initial stress and for the different temperatures, in Fig. 6b. As for the case of Zircaloy-4, it is clear that C' depends, in general, both on stress and on temperature. This is illustrated by the broken straight lines that give the approximate stress dependence

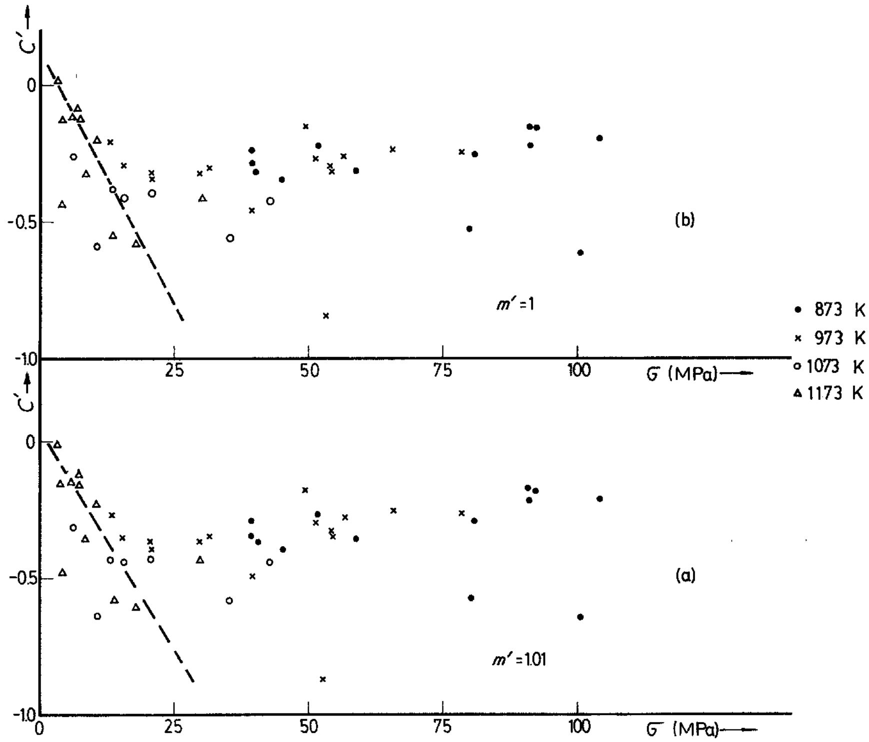

Figure 4 C' for Zircaloy-4 as a function of stress, for the different temperatures, as calculated with Equation 2 for each individual set of data. (a) $m' = 1.01$; (b) $m' = 1$.

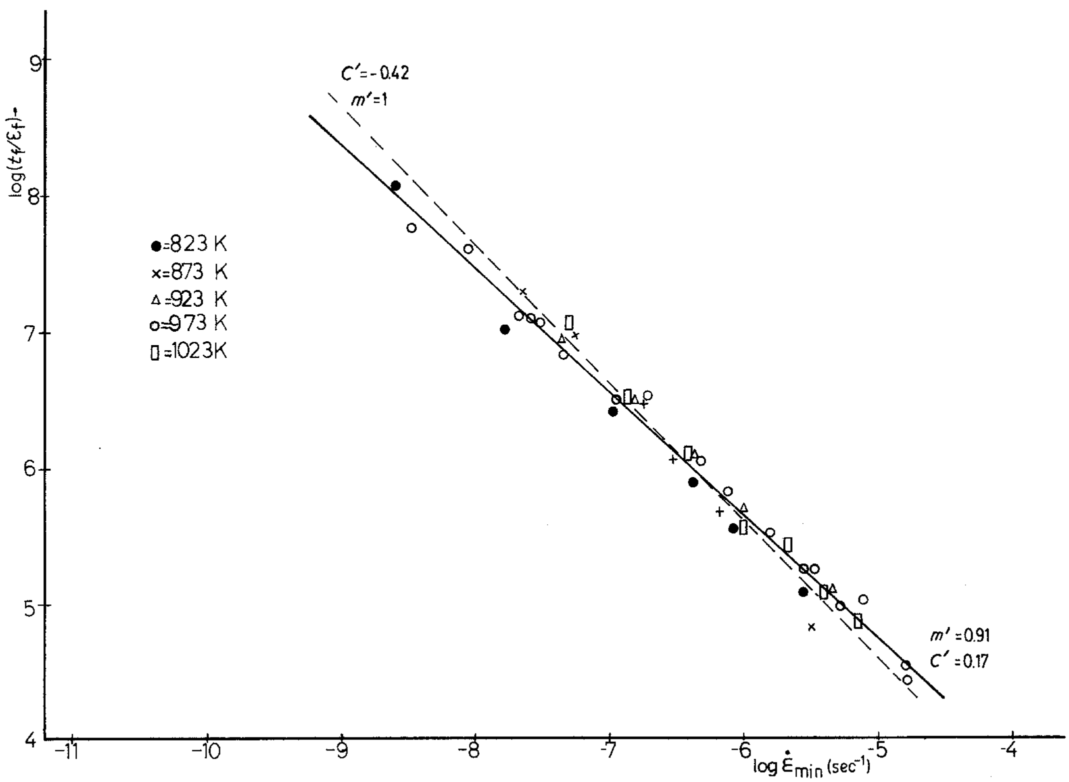

Figure 5 MMGR plot for constant load creep data obtained in SS AISI 304. The broken straight line corresponds to $m' = 1$.

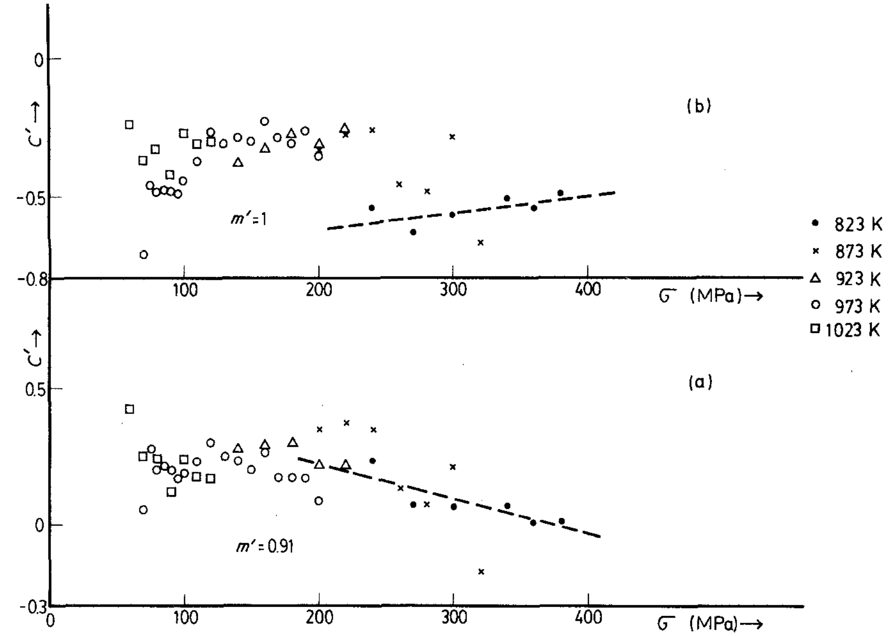

Figure 6 C' for SS AISI 304 as a function of stress, for different temperatures, as calculated using Equation 2 for each individual set of data. (a) $m' = 0.91$; (b) $m' = 1$.

of C' for the data obtained at 823 K. The MMGR plot for $m' = 1$, leading to $C' = -0.42$, is shown by the broken straight line of Fig. 5.

4. Discussion

From the results just described it can be concluded that C' is not strictly constant even if a $\log(t_f/\epsilon_f)$–$\log \dot{\epsilon}_{min}$ representation leads to a roughly linear plot. In addition, the experimental data can be described, within an equivalent limit of accuracy, both by Equation 2 and by Equation 8 ($m' = 1$). This is evident for the data obtained in Zircaloy-4, shown in Fig. 3. For the data in SS AISI 304, shown in Fig. 5, both the straight line with slope $m' = 0.91$ and that with slope $m' = 1$ give almost the same degree of accuracy for the description of the experimental results. Furthermore, on comparing Figs. 6a and b it can be seen that a lower dispersion in C' is observed for $m' = 1$. A similar situation is found for the data reported in [2] where for Alloy A5 the MMGR plot shows less dispersion than for Alloy A6, with the value $m' = 0.99$ for the first and $m' = 0.92$ for the second. It can be shown easily from Fig. 4 of [2] that Alloy A6 can be represented within the same degree of accuracy by Equation 8.

Unfortunately, only MMGR diagrams for these two alloys were reported, but from Table II of [2] it can be seen that the fitting to Equation 2, of data obtained in various alloys tends to give a lower standard deviation when $m' \simeq 1$. Furthermore, $C' < 0$, i.e. $K < 1$, when $m' \simeq 1$ and for only three alloys the authors reported $C' > 0$ with $m' \simeq 0.9$.

In summary, it is clear that the data considered are described roughly by Equation 6. The results are approximately described by the MMGR representation since the changes in K, from one creep curve to another, are smoothed-out by the double logarithmic representation. In fact, as shown by Figs. 5 and 6, C' changes substantially and gradually with stress and temperature, showing that K is not strictly constant.

A good fitting to Equation 8 is expected if the $\dot{\epsilon}$–t curves have a strong contribution from the steady-state creep. In fact, if it is assumed that the creep rate is expressed, up to fracture, by

$$\dot{\epsilon} = k\sigma^p F(T) \qquad (9)$$

where k and p are constants and $F(T)$ is a function only of temperature, then, since $d\epsilon = \dot{\epsilon}\, dt$, for an experiment performed at constant stress

$$\epsilon_f = \dot{\epsilon}_{min} t_f \qquad (10)$$

so that $m' = 1$ and $C' = 0$.

If the experiment is performed at constant load then

$$\sigma = \sigma_0 \exp(\epsilon) \qquad (11)$$

where σ_0 is the initial stress and ϵ the instantaneous strain. Then

$$\mathrm{d}\epsilon = k(\sigma_0)^p \exp(p\epsilon)F(T)\,\mathrm{d}t \quad (12)$$

and

$$\int_0^{\epsilon_f} \exp(-p\epsilon)\,\mathrm{d}\epsilon = k(\sigma_0)^p F(T) t_f \quad (13)$$

which, after integration and rearranging terms, leads to

$$\epsilon_f = -\frac{1}{p}\ln(1-\dot{\epsilon}_{min} p t_f) \quad (14)$$

where

$$\dot{\epsilon}_{min} = k(\sigma_0)^p F(T) \quad (15)$$

If $\dot{\epsilon}_{min} p t_f \ll 1$, Equation 14 reduces to $\epsilon_f = \dot{\epsilon}_{min} t_f$ so that $C' = 0$ and $m' = 1$.

It should be pointed out that the curves shown in Fig. 1 are only given to illustrate the problem and the considerations made in the paper are valid whenever it is possible to measure ϵ_f, t_f and $\dot{\epsilon}_{min}$. No hypotheses are introduced about the mechanisms controlling creep. In cases where intergranular creep fracture damage, for example, can accumulate in a way which is independent of the development of bulk strain, $\dot{\epsilon}_{min}$ and t_f can be easily measured and ϵ_f can be obtained from fiducial marks in the specimen. The considerations made in the paper apply also to this case since the MMGR relationship can be used without knowing the complete $\dot{\epsilon}$–t curve.

Finally, according to Toscano and Boček [3] the MMGR relationship can be derived from a high temperature failure model developed by Edward and Ashby [4]. The results of their calculations were compared with creep experiments performed on Zircaloy-2, Zircaloy-4 and SS AISI 304, and according to the model, $m' = 1$ and $C' \ll 1$. In addition, C' is independent of stress and temperature. For the Zircaloy-4 data shown in Fig. 3, which were also considered by these authors, it was shown that C' depends upon stress and temperature. The same considerations are also valid for the data on SS AISI 304 shown in Fig. 5. It is clear that any physical model that pretends to describe the MMGR relationship within the accuracy considered in this paper should be taken with caution. Furthermore, within these limits it is difficult to give a physical significance to the MMGR relationship.

5. Conclusions

For the experimental data reported in the literature, both the Monkman–Grant and the modified Monkman–Grant relationships are the result of a rough approximation of the integral under the strain rate–time curves and of a smoothing-out of the scatter in the results due to the double-logarithmic representation. Furthermore, these relationships can only be used for an order of magnitude estimate of any one of the quantities ϵ_f, t_f and $\dot{\epsilon}_{min}$ when the other two are known.

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